

## LOWER-DIMENSION VACUUM DEFECTS IN LATTICE YANG-MILLS THEORY

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We overview lattice data on  $d = 1, 2, 3$  vacuum defects in four-dimensional gluodynamics. In all the cases defects have total volume which scales in physical units (with zero fractal dimension). In case of  $d = 1, 2$  the defects are distinguished by ultraviolet divergent non-Abelian action as well. This sensitivity to the ultraviolet scale allows to derive from the continuum theory strong constraints on the properties of the defects. The constraints turn to be satisfied by the lattice data. In the  $SU(2)$  case we introduce a classification scheme of the defects which allows to (at least) visualize the defect properties in a simple and unified way. Not-yet-checked relation of the defects to the spontaneous chiral symmetry breaking is suggested by the scheme.

### 1. Introduction

Somehow, it went largely unnoticed that non-perturbative QCD has been changing fast. The change is mostly due to results of lattice simulations which ask sometimes for novel continuum-theory models, see, in particular, <sup>1</sup>. Moreover, in many cases the lattice results are formulated in specific language and do not allow for any immediate interpretation in terms of the continuum theory. In this way, there arises a mismatch between richness of the lattice data and scarceness of their interpretation in the continuum theory.

An important example of this type are models of confinement. Indeed, instantons still dominate thinking on non-perturbative physics on the continuum-theory side. On the other hand, it is known from the lattice measurements that the instantons do not confine <sup>2</sup>. Moreover, the vacuum fluctuations which are responsible for the confinement have been also identified and turn to be monopoles and P-vortices, for review see, e.g., <sup>3,4</sup>.

There is no understanding whatsoever of these confining fluctuations in

terms of the continuum theory. Moreover, if now someone decides to go into interpretation of the lattice data there is no regular way to approach the problem. The point is that the monopoles and vortices are defined on the lattice rather algorithmically, than directly in terms of the gluonic fields. The central step is the use of projections which replace, say, the original non-Abelian fields by the closest Abelian-field configurations. The projection is a highly nonlocal procedure defined only on the lattice and blocks out any direct interpretation of the data.

To circumvent this difficulty, we attempt to summarize here the lattice data on the confining vacuum fluctuations entirely in terms of continuum theory. Hopefully, this could facilitate appreciation of the results of the lattice simulations. In particular, we emphasize that the confining fluctuations appear to be vacuum defects of dimension lower than  $d = 4$ .

What challenges the continuum theory in the most direct way is a relatively recent discovery that the monopoles, see <sup>5</sup> and references therein, and vortices, see <sup>6</sup>, are associated with ultraviolet divergent non-Abelian action. Since gluodynamics is well understood at short distances, this newly discovered sensitivity of the vacuum defects to the ultraviolet scale makes them subject to strong constraints from the continuum theory <sup>7</sup>.

The presentation is as follows. In Sect. 2 we summarize lattice data on the confining fluctuations. In Sect. 3 constraints from the continuum theory are outlined. In Sect. 4 possible relation to dual formulations of the Yang-Mills theories is discussed.

## 2. Lattice phenomenology

### 2.1. Total volume

Imagine that indeed there exist low-dimension structures in the vacuum state of gluodynamics. Which  $SU(2)$  invariants could be associated with such defects? First of all, we could expect that the total volume of the corresponding defects scales in physical units. What this means, is easier to explain on particular examples.

*$d = 0$  defects.* In this case, we are discussing density of points in the  $d = 4$  space. And the expectation for the total number of the point-like defects would be

$$N_{tot} = c_0 \Lambda_{QCD}^4 \cdot V_4, \quad (1)$$

where  $V_4$  is the volume of the lattice and  $\Lambda_{QCD}$  can be understood either as a position of the pole of the (perturbative) running coupling or, say,

as  $\Lambda_{QCD} = \sqrt{\sigma_{SU(2)}}$  where  $\sigma_{SU(2)}$  is the string tension. Appearance of  $\Lambda_{QCD}$  in (1) would signal relevance of the fluctuations to the confinement.

Equation (1) could be readily understood if we were discussing number of instantons. However, it might worth mentioning from the very beginning that the instantons actually do not belong to the sequence of the vacuum fluctuations which we are going to consider.

$d = 1$  *defects*. The  $d = 1$  defects are lines. For the total length one can expect

$$L_{tot} = c_1 \Lambda_{QCD}^3 \cdot V_4 . \quad (2)$$

Such defects can be identified with the **percolating monopoles** (for latest data see <sup>8,9</sup>). Percolation <sup>a</sup> means that there exists a large cluster of monopole trajectories stretching itself through the whole volume of the lattice. In the limit of infinite volume the percolating cluster also becomes infinite. Note also that the monopole trajectories are closed by definition, as a reflection of the monopole charge conservation.

It is worth emphasizing that the scaling law (2) is highly nontrivial and, from the point of view of the lattice measurements, represents a spectacular phenomenon. Indeed, on the lattice one changes arbitrarily the lattice spacing,  $a$  while the corresponding bare coupling,  $g(a)$  is changed logarithmically, according to the renormgroup. The scaling law (2) implies that the probability  $\theta(a)$  for a given link (actually, on the dual lattice) to belong to the percolating cluster is changing as a power of  $a$ :

$$\theta(a) \sim (\Lambda_{QCD} \cdot a)^3 . \quad (3)$$

Thus, in this case powers, and not logs, of the ultraviolet cut off are observed. In other words, there is no perturbative background to the defects which we are discussing and we are addressing directly non-perturbative physics.

$d = 2$  *defects*. The defects are now two-dimensional surfaces and for the total area the scaling law would read:

$$A_{tot} = c_2 \Lambda_{QCD}^2 \cdot V_4 . \quad (4)$$

Such defects can be identified with **percolating P-vortices** which are known to satisfy (4) <sup>4</sup>, for the latest data see <sup>6</sup>. As in all other cases discussed here the evidence is pure numerical, though. Because of the space considerations we do not discuss here error bars, concentrating only on the general picture. Details can be found in the original papers.

<sup>a</sup>For theoretical background see, e.g., <sup>10</sup>.

$d = 3$  defects. For a percolating three-dimensional volume we could expect:

$$V_3 = c_3 \Lambda_{QCD} \cdot V_4 . \quad (5)$$

First indications on existence of such defects were obtained recently <sup>11</sup>.

## 2.2. Non-Abelian action associated with the defects

Defects can be distinguished by their non-Abelian action as well. Since we have not specified yet the defects dynamically, at first sight, we cannot say anything on their action. Surprisingly enough, there exist educated guesses concerning the non-Abelian action of the defects based on their dimension alone.

$d=1$  defects. This case is singled out by the consideration that trajectories correspond to particles. Particles, on the other hand, belong to field theory and we may hope to get insight into the properties of the trajectories from field theory. And, indeed, the action

$$S = M \cdot L , \quad (6)$$

where  $L$  is the length of the trajectory and  $M$  is a mass parameter coincides with the classical action for a free particle of mass  $M$ . One may hope, therefore, that by evaluating propagation of a particle as a path integral with the action (6) one reconstructs the quantum propagator of a free particle. And, indeed, this theoretical construction works. Moreover, it is well known as the so called polymer approach to field theory, see, in particular, <sup>12</sup>. Note that the use of the Euclidean (rather than Minkowskian) space is actually crucial to evaluate the corresponding path integral. Also, one needs to introduce lattice to formulate the theory.

Although the use of the action (6) does allow to recover the free field propagator, the **propagating mass** turns to be not the same  $M$  but is equal to

$$m_{prop}^2 = \frac{(const)}{a} \left( M(a) - \frac{\ln 7}{a} \right) , \quad (7)$$

where the constants  $const, \ln 7$  are of pure geometrical origin and depend on the lattice used. In particular,  $\ln 7$  corresponds to the hypercubic lattice.

Note that in Eq (7) we reserved now for dependence of the mass parameter  $M(a)$  on the lattice spacing  $a$ . Indeed, tuning of  $M(a)$  to  $\ln 7/a$  is needed to get an observable mass (7) independent on the lattice spacing  $a$ .

Thus, our prediction for the action associated with  $d = 1$  defects (which are nothing else but the monopole trajectories) is that the action is close to

$$S_{mon} \approx \frac{\ln 7}{a} \cdot L . \quad (8)$$

Indeed, in this way we can explain that the length of the trajectories does not depend on  $a$ , see Eq (2).

Prediction (8) does agree with the results of direct measurements of the non-Abelian action of the monopoles <sup>5</sup>. Let us emphasize that the prediction (8) does not use anything specific for monopoles and is rooted in the standard field theory. Indeed, the polymer approach to field theory is no better no worse than other approaches.

$d = 2$  defects. There exists simple theoretical argumentation in favor of an ultraviolet divergent action of the two-dimensional defects (or vortices) as well. Consider the so called gluon condensate

$$\langle (G_{\mu\nu}^a)^2 \rangle \approx \frac{N_c^2 - 1}{a^4} (1 + O(\alpha_s)) , \quad (9)$$

where  $G_{\mu\nu}^a$  is the non-Abelian field strength tensor and  $a$  is the color index. Note that the condensate (9) on the lattice is in fact nothing else but the average plaquette action <sup>14</sup>.

The vacuum expectation value (9) diverges as the fourth power of the ultraviolet cut off. This divergence is in one-to-one correspondence with the divergence in the density of the vacuum energy, well known in the continuum theory. If one neglects interaction of the gluons, the gluon condensate (9) reduces to a sum over energies of the zero-point fluctuations. That is why the r.h.s. of (9) in the zero approximation is proportional to the number of degrees freedom, that is to the number of gluons. Accounting for the gluon interaction brings in perturbative corrections. For details and most advanced calculations see the last paper in Ref. <sup>14</sup>.

Note now that the monopole trajectories with the properties (2) and (8) give the following contribution to the the gluon condensate:

$$\langle (G_{\mu\nu}^a)^2 \rangle_{mon} \approx \frac{N_c^2 - 1}{a^4} (const) (\Lambda_{QCD} \cdot a)^3 . \quad (10)$$

In other words, monopoles correspond to a power like correction to the perturbative value of the gluon condensate.

Now, the central point is that there exists rather well developed theory of the power corrections, for review see <sup>15</sup>. The leading power-like correction

is expected to be associated with the so called ultraviolet renormalon. The corresponding contribution is of order

$$\langle (G_{\mu\nu}^a)^2 \rangle_{uv-ren} \approx \frac{N_c^2 - 1}{a^4} (const) (\Lambda_{QCD} \cdot a)^2 . \quad (11)$$

Note that the power of  $(\Lambda_{QCD} \cdot a)$  here is different from what the monopoles give, see Eq (10).

From this point of view it would be unnatural if the monopoles exhausted the power corrections. Moreover, if the  $d = 2$  defects with the total area satisfying (4) have the action

$$S_{vort} = (const) \frac{A_{vort}}{a^2} , \quad (12)$$

then their contribution would fit the ultraviolet renormalon (11). In this way one could have predicted (12). The data on the non-Abelian action associated with the P-vortices <sup>6</sup> do confirm the estimate (12):

$$S_{vort} \approx 0.54 \frac{A_{vort}}{a^2} , \quad (13)$$

where  $A_{vort}$  is the total area of the vortices.

$d = 3$  defects. Proceeding to the three dimensional defects we could predict that the story does **not** repeat itself and there is no ultraviolet action associated with the  $V_3$ . Indeed, if the action would be of order  $V_3/a^3$ , the corresponding power correction to the gluon condensate would exceed the ultraviolet renormalon, see (11), by  $(\Lambda_{QCD} \cdot a)^{-1}$  and contradict the theory.

The data <sup>11</sup> indeed do not indicate any excess of the action associated with the  $V_3$ . One can claim this to be a success of the theory. It is also true, however, that the three dimensional defects so far are lacking identity in terms of gauge invariant characteristics.

### 2.3. Entropy associated with the defects

The ultraviolet divergences in the action of the monopoles, see (8), and of vortices, see (12), suggest, at first sight, that these fluctuations are not physical and exist only on scale of the lattice spacing  $a$ . Indeed, the probability, say, to observe a monopole trajectory of length  $L$  is suppressed by the action as

$$W(L) \approx \exp(-S_{mon}) \cdot (Entropy) \sim \exp(-\ln 7L/a) \cdot (Entropy) , \quad (14)$$

and in the continuum limit of  $a \rightarrow 0$  the suppression due to the action is infinitely strong. Since, on the other hand, the observed length of the

monopole trajectories does not tend to zero in the limit  $a \rightarrow 0$  the suppression by action is to be canceled by the same strong *enhancement* due to the entropy. Let us discuss this issue in more detail in cases  $d = 1, 2$ .

*d=1 defects.* In this case, the entropy factor is the number  $N(L)$  of different trajectories of same length  $L$ . It is quite obvious that for  $L$  fixed and  $a \rightarrow 0$ ,  $N(L)$  grows exponentially with  $L$ <sup>13</sup>. Indeed, trajectory on the lattice is a sequence of steps of the length  $a$ . The number of steps is  $L/a$  and at each step one can arbitrarily choose direction. The number of directions is determined by the geometry of the lattice. This is the origin of the factor  $\ln 7/a$  in Eq (7) valid for the hypercubic lattice.

Therefore, the observation (8) is nothing else but the statement that in case of the monopole trajectories the entropy is fine tuned to the action. Moreover, since there is actually no free parameter in the theory (the QCD coupling is running and cannot be tuned) we are dealing rather with **self tuning** of the monopole trajectories. This should be a dynamical phenomenon. The author of the present review finds this observation of self-tuned objects on the lattice absolutely remarkable.

*d=2 effects.* The suppression due to the action is again there, see (13). Moreover, from lattice measurements one can conclude that the action is not simply proportional to the total area of the vortex but depends also on local geometrical properties of the surface, for detail see<sup>16</sup>.

As for the entropy, the lattice measurements indicate<sup>16</sup> that mutual orientation of adjacent plaquettes belonging to the vortices is random. This observation, in turn, implies exponential enhancement of the entropy of the surfaces.

Theoretically, one can determine (from indirect argument) entropy of P-vortices in case of  $Z_2$  gauge theory. In case of the  $Z_2$  theory plaquettes take on values  $\pm 1$  and the P-vortices by definition pierce the negative plaquettes. Numerically,

$$(Entropy)_{Z_2-vortices} \approx \exp(0.88A_{tot}/a^2) , \quad (15)$$

where  $A_{tot}$  is the total area of the negative plaquettes.

We see that the action (13) is *not* tuned to the entropy of  $Z_2$ -vortices (15). A straightforward interpretation of this observation is that the entropy of the P-vortices in SU(2) gluodynamics is lower than that in case of the  $Z_2$  theory. As we will discuss later, the P-vortices in the gluodynamics case are populated by magnetic monopoles. The lower value of the entropy could be understood as evidence that at short distances it is rather monopoles which are to be considered as fundamental while the

P-vortices ‘follow’ the monopole trajectories. This issue deserves further detailed study and discussion.

#### 2.4. *Alignment of geometry and non-Abelian fields*

So far we considered geometry and non-Abelian fields associated with the defects separately. The total volume is a geometrical scalar and the total action is a scalar constructed on the fields. Of course, observation of invariant characteristics which mix up geometry and fields would be even more interesting.

*d=3 defects.* Let us start with  $d = 3$  and assume existence of three dimensional volumes to be granted. The volume percolates on the scale  $\Lambda_{QCD}^{-1}$ . However, if we tend  $a \rightarrow 0$  then we can think in terms of large (in the lattice units)  $d = 3$  defects. What are possible  $SU(2)$  invariants associated with these volumes?

In  $d = 3$  the  $SU(2)$  gluodynamics is described by three vectors  $\mathbf{H}^a$  ( $a=1,2,3$ ) where the ‘magnetic fields’  $\mathbf{H}^a$  are vectors both in the coordinate and color spaces. Generically, there are three independent vectors and the simplest invariant constructed on these fields is their determinant. However, the absolute value of the determinant varies from one point of the  $d = 3$  volume to another. The invariant which can be associated with the whole volume is, obviously, the sign of the determinant <sup>b</sup> :

$$I_3 = \text{sign}\{\epsilon^{ikl}\epsilon_{abc}H_i^a H_k^b H_l^c\} . \quad (16)$$

Unfortunately, there are no lattice data which could confirm or reject this prediction.

*d = 2 defects.* The invariant (16) is uniquely defined as far as all three magnetic fields are indeed independent. If there are only two independent vectors the determinant has zero of first order and geometrically we have then a closed  $d = 2$  surface, as a boundary of the  $d = 3$  defects. It seems natural to speculate that these boundaries are our  $d = 2$  defects, or vortices.

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<sup>b</sup>The classification scheme of the defects we are proposing below, to our knowledge, has not been discussed in literature (an earlier version can be found in Ref. <sup>18</sup>). However, at least partially, it is close to or motivated by well known papers, see, in particular, <sup>17</sup>. It is worth emphasizing that we are using the continuum-theory language, assuming fields to be continuous functions of the coordinates. Since on the lattice the measurements are performed on the scale of the lattice spacing  $a$ , the lattice fields fluctuate wildly on the same scale. Thus, the underlying assumption is that the continuum-theory language is still valid on average, so that the perturbative fluctuations do not interfere with topology.



Thus, we come to the prediction that the percolating  $d = 2$  surfaces and non-Abelian fields are aligned with each other. In other words, the non-Abelian fields associated with the vortices and resulting in the excess of the action (12) are predicted to spread over the surface while the perpendicular component is to vanish. This prediction works <sup>6</sup> *perfectly well*.

$d = 1$  *defects*. The next step is an reiteration of the previous one. Namely, there could be zeros of the second order of the determinant (16) and geometrically zeros of the second order are closed trajectories. Moreover, if these closed lines are monopole trajectories (which are indeed closed by definition), the non-Abelian field of the monopoles is to be aligned with their trajectories. This expectation *agrees* with the existing data <sup>19</sup> as well.

It is amusing that the monopoles are expected to be locally Abelian. Indeed, there is only one independent color magnetic field associated with zeros of second order of the determinant (16). Moreover, they are singular, see (8). Thus, we are coming to an after-the-fact justification of the use of the Abelian projection to detect the monopoles. On the other hand, their field is absolutely not spherically symmetrical (which would be the case for the Dirac monopoles). And this spatial asymmetry is manifested in the measurements <sup>19</sup>.

## 2.5. Spontaneous breaking of chiral symmetry

Note that our classification scheme predicts that the  $d = 3$  defects are characterized by invariants which distinguish between left- and right- hand coordinates. Of course, on average the regions with determinants (16) positive and negative occupy the same volume. (Moreover, in the continuum limit the the percolating  $d = 3$  volume, see Eq (5), occupies a vanishing part of the whole  $d = 4$  space).

As is mentioned above there are no direct measurements of the sign of the determinant (16) within the  $d=3$  defects. However, it is known that removal of the P-vortices restores chiral symmetry <sup>20</sup>. Also, there exists independent evidence that some 3d volumes are related to the spontaneous symmetry breaking in case of  $SU(3)$  gluodynamics <sup>21</sup> <sup>c</sup>. However, it seems too early to speculate that the two 3d volumes in question are actually the same.

<sup>c</sup>An obvious generalization of our invariant (16) to the  $SU(3)$  case would be  $sign \{f_{abc}\epsilon^{ikl}H_i^a H_k^b H_l^c\}$ .

### 3. Theoretical constraints

#### 3.1. *Consistency with the asymptotic freedom*

It is worth emphasizing that there is no developed theory of the defects considered. So far we summarized lattice observations. Moreover, we have been even avoiding discussion how the defects are defined and observed on the lattice, for reviews see, e.g., <sup>3,4,22</sup>. Let us only mention that the guiding principle to define the defects was the search for effective infrared degrees of freedom responsible for the confinement.

However, what is most amusing from the theoretical point of view is that the defects have highly non-trivial properties in the ultraviolet. The ultraviolet divergence in the action, see (8), (13) are most remarkable.

Indeed, Yang-Mills theories are well understood at short distances. The only divergence which is allowed on the fundamental level is that one in the coupling  $\alpha_s$ . Thus, one could argue that all the ultraviolet divergences are calculable perturbatively in asymptotically free theories. Moreover, this statement seems rather trivial. What is actually not so trivial is that on the lattice one can also consider power-like divergences, see, e.g., (10), (11). In the continuum theory, power-like ultraviolet divergences usually are used, at best, for estimates. On the lattice, the ultraviolet cut off is introduced explicitly and one can treat power-like divergent observables in a fully quantitative way, see, e.g., (9). This extends in fact the predictive power of the theory.

It is, therefore, no surprise that using the asymptotic freedom one can derive strong constraints on the properties of the vortices <sup>7</sup>.

#### 3.2. *Classical condensate $\langle \phi_{magn} \rangle$*

Let us consider first the vacuum condensate of the magnetically charged field  $\langle 0 | \phi_{magn} | 0 \rangle$ . Of course, in the YM theory there is no fundamental magnetically charged field. However, the monopole trajectories are observed on the lattice. Using the polymer approach to field theory we can translate the lattice data on trajectories into a field-theoretic language, see Ref <sup>23,7</sup> and references therein. The only assumption is that there exists an effective field theory for the magnetically charge field.

In particular, the percolating cluster corresponds to the classical vacuum expectation value  $\langle \phi \rangle$ . One can derive <sup>23</sup>:

$$\langle \phi_{magn} \rangle^2 \approx \frac{a}{8} \rho_{perc} \approx (const) \Lambda_{QCD}^2 (a \cdot \Lambda_{QCD}) . \quad (17)$$

### 3.3. Vacuum expectation value $\langle |\phi_{magn}|^2 \rangle$

One can also expect that there exist quantum fluctuations. And indeed, apart from the percolating cluster, there observed finite monopole clusters. Which are naturally identified with the quantum fluctuations. A basic characteristic for these clusters is again their total length. By definition:

$$L_{tot} \equiv L_{perc} + L_{fin} \equiv \rho_{perc} \cdot V_4 + \rho_{fin} \cdot V_4 , \quad (18)$$

where  $\rho_{perc}, \rho_{fin}$  are called the densities of the percolating and finite monopole clusters, respectively.

Using the polymer approach to the field theory one can express the vacuum expectation value of the magnetically charged field in terms of the monopole trajectories <sup>23</sup>:

$$\langle |\phi|^2 \rangle = \frac{a}{8} (\rho_{perc} + \rho_{fin}) . \quad (19)$$

Instead of deriving this relation (which is also quite straightforward) let us explain why (19) is natural. Concentrate on the quantum fluctuations, that is on  $\rho_{fin}$ . Moreover, consider small clusters  $L \ll \Lambda_{QCD}^{-1}$ . Then there is no mass parameter at all and on pure dimensional ground one would expect:

$$(\rho_{fin})_{dimension} = \frac{const}{a^{d-1}} = \frac{const}{a^3} \quad (20)$$

where  $d$  is the number of dimensions of space and we consider the  $d = 4$  case. If this dimensional estimate held, then the vacuum expectation value would be quadratically divergent in the ultraviolet:

$$\langle |\phi|^2 \rangle_{dimension} \sim a^{-2} . \quad (21)$$

And we rederive the standard quadratic divergence in the vacuum expectation value of a scalar field.

Now, the central point is that although we call all these estimates ‘dimensional’ or ‘natural’ we are not allowed to have (21). Indeed, Eq. (21) would hold for an *elementary* scalar field. However, we are not allowed to have new elementary particles at short distances. Because of the asymptotic freedom, there are only free gluons at short distances. And what we are allowed to have for the vacuum expectation value in point? Clearly:

$$\langle |\phi|^2 \rangle_{allowed} \sim \Lambda_{QCD}^2 . \quad (22)$$

In terms of the monopole trajectories (which are our observables) Eq (22) reduces to:

$$\rho_{fin} \sim \frac{\Lambda_{QCD}^2}{a} . \quad (23)$$

It is most remarkable that the data <sup>8,9</sup> do comply with (23)!

### 3.4. *Branes*

It is of course very gratifying that the data comply with the constraint (22). On the other hand, the reader may feel that our summary of the phenomenology looks self contradictory. Indeed, first we observed that the non-Abelian monopole action corresponds to a point-like particle, see (8). But then we said that there should be no new particles, and the data agree with that constraint.

Still, there is no contradiction between these observations. Rather, taken together they amount to observation of a new object, which can be called branes. Indeed, cancellation of  $\ln 7/a$  in the equation for the mass is needed to balance the entropy at very short distances of order  $a$ . Eq (22), on the other hand, is a global constraint. The geometrical meaning of this constraint is actually transparent. Namely, it means that on large scale the monopoles live not on the whole  $d = 4$  space but on its  $d = 2$  subspace.

Within the classification scheme of the defects which we discussed above, this association of the monopoles with surfaces is automatic. Algorithmically, however, the trajectories and surfaces are defined independently. And the fact that the monopole trajectories do belong to surfaces is highly non-trivial from the observational point of view.

Thus, what is observed on the lattice are  $d = 2$  surfaces populated with “particles” (better to say with the tachyonic mode of the monopole field). When we call this objects branes we emphasize that the affinity of the monopoles to the surfaces remains true even at the scale  $a$ . Traditional discussions of the P-vortices and monopoles emphasize, on the other hand, physics in infrared, and one talks about ‘thick vortices’, see, e.g. <sup>4</sup>.

### 3.5. *Implications for models*

*Abelian Higgs model.* The lattice data on monopoles are usually interpreted in terms of an effective Abelian Higgs model, see in particular <sup>24</sup> and references therein. Our Eq (17) implies, however,

$$\langle \phi_{magn} \rangle \sim (a \cdot \Lambda_{QCD})^{1/2} \Lambda_{QCD} \quad (24)$$

It is most remarkable that the classical condensate vanishes in the continuum limit  $a \rightarrow 0$ . Nevertheless, the heavy quark potential at large distances generated by the monopoles remains the same since it is determined entirely by  $\rho_{perc}$  which scales in the physical units!

To establish a relation to the standard fit to the Abelian Higgs model one should use, most probably, matching of the two approaches at some  $a$ <sup>d</sup>.

*Gauge invariant condensate of dimension two.* Eqs (22), (23) provide us with a value for a gauge invariant condensate of dimension two:

$$\langle |\phi_{\text{magn}}|^2 \rangle \approx a \cdot \rho_{\text{fin}} \sim \Lambda_{QCD}^2. \quad (25)$$

In terms of the fundamental variables, condensate of dimension two was introduced<sup>25</sup> as the minimum value along the gauge orbit of the gauge potential squared  $(A_\mu^a)^2_{\text{min}}$ . While the  $\langle (A_\mu^a)^2 \rangle_{\text{min}}$  is contaminated with perturbative divergences, the condensate (25) provides us, on the phenomenological level, directly with a non-perturbative condensate of dimension two. Existence of a gauge invariant condensate of dimension two is crucial for models of hadrons, see, in particular,<sup>26</sup>. On theoretical side, the nonperturbative part of the condensate of dimension two in the Hamiltonian picture is related to the Gribov horizon<sup>27</sup>.

*Percolation.* The fact that the monopole action is ultraviolet divergent, see (8), allows to consider them as point-like and apply percolation theory. In particular, one derives in this way the spectrum of finite monopole clusters as function of their length  $L_{\text{fin}}$ :

$$N(L_{\text{fin}}) \sim \frac{1}{L_{\text{fin}}^3}, \quad (26)$$

which agrees with the data, for details see<sup>23</sup>.

Moreover, it is a common feature of percolating systems that near the phase transition, in the supercritical phase, the density of the percolating cluster is small. Then the observation (3) can be interpreted as indication that in the limit  $a \rightarrow 0$  we hit the point of second-order phase transition.

Note also that second order phase transition is associated usually with a massless excitation. At first sight, there is no massless excitation in our case, however. The resolution of the paradox is that percolation (or randomization) happens now on the ultraviolet scale. Respectively, all the masses are to be measured in the lattice units,  $1/a$ . In this sense a glueball mass,

$$m_{\text{glueball}}^2 \sim \frac{1}{a^2} \cdot (a \cdot \Lambda_{QCD})^2, \quad (27)$$

<sup>d</sup>The author would like to acknowledge discussion of this point with M.N. Chernodub

in the limit  $a \rightarrow 0$  corresponds to a massless excitation. Moreover, one expects that the glueball mass controls the scale of fluctuations of the monopole clusters. But these expectations have not been checked yet (see, however, <sup>28</sup>).

*Singular fields.* Although we are repeatedly emphasizing that the branes are associated with singular fields it should also be mentioned that the non-Abelian fields considered are significantly smaller than the corresponding projected fields. In particular, the Dirac monopole would have a larger action:

$$S_{Dir} \sim \frac{1}{g^2} \cdot \frac{L}{a} \gg \ln 7 \frac{L}{a} \quad (28)$$

where  $g^2$  is the gauge coupling and we consider the limit  $g^2 \rightarrow 0$ . Already at presently available  $g^2$  the action calculated in terms of the projected Abelian fields (corresponding to the Dirac monopole) is a few times larger than the actual non-Abelian action which determines the dynamics and which we discussed so far. The same is true for the vortices.

This distinction – in terms of the action – between the Dirac and lattice monopoles is very important from theoretical point of view. The observed monopoles are associated with singular non-Abelian fields but these fields are no more singular than ordinary zero-point fluctuations, or perturbative fields. The Dirac monopoles, on the other hand, in the limit of  $g^2 \rightarrow 0$  would be more singular than the perturbative fields. According to the standard ideas of the lattice theories, such fields could actually be removed without affecting the basic physical content of the theory. The lattice monopoles and vortices, on the other hand, cannot be removed without affecting perturbative fluctuations as well. In the next two sections we will consider this issue in more detail.

#### 4. Towards duality

It is a well known that topological excitations of a ‘direct’ formulation of a theory may become fundamental variables of a dual formulation of the same theory. Examples can be found, e.g., in the review <sup>29</sup>. Little, if anything, known theoretically on the dual formulation of the Yang-Mills theories without supersymmetry. Nevertheless, generically one might think in terms of branes <sup>30</sup>. In case of supersymmetric extensions of YM theories the branes are classical solutions. One could speculate that the branes discussed in this review are ‘quantum branes’. Of course, it remains a pure speculation until something definite could be said on the properties of the

quantum branes on the theoretical side. It is amusing, however, that there is a sign of duality between the branes discussed in the preceding section and high orders of perturbation theory<sup>31</sup>.

## 5. Long perturbative series

### 5.1. *Expectations*

Let us start with reminding the reader some basic facts about perturbative expansions, for detailed reviews see, e.g.,<sup>15</sup>. A generic perturbative expansion for a matrix element of a local operator looks as:

$$\langle O \rangle = (parton\ model) \cdot \left(1 + \sum_{n=1}^{\infty} a_n \alpha_s^n\right), \quad (29)$$

where we normalized the anomalous dimension of the operator  $O$  to zero and  $\alpha_s$  is small,  $\alpha_s \ll 1$ .

In fact, expansions (29) are only formal since the coefficients  $a_n$  grow factorially at large  $n$ :

$$|a_n| \sim c_i^n \cdot n!, \quad (30)$$

where  $c_i$  are constants. Moreover, there are a few sources of the growth (30) and, respectively,  $c_i$  can take on various values. The factorial growth of  $a_n$  implies that the expansion (29) is asymptotic at best. Which means, in turn, that (29) cannot approximate a physical quantity to accuracy better than

$$\Delta \sim \exp(-1/c_i \alpha_s) \sim \left(\Lambda_{QCD}^2 \cdot a^2\right)^{b_0/c_i}, \quad (31)$$

where  $b_0$  is the first coefficient in the  $\beta$ -function. To compensate for these intrinsic uncertainties one modifies the original expansion (29) by adding the corresponding power corrections with unknown coefficients.

In case of the gluon condensate the theoretical expectations can be summarized as:

$$\langle 0 | \frac{-\beta(\alpha_s)}{\alpha_s b_0} (G_{\mu\nu}^a)^2 | 0 \rangle \approx \alpha_s \frac{(N_c^2 - 1)}{a^4} \left(1 + \sum_{n=1}^{\sim N_{ir}} a_n \alpha_s^n + (const) a^4 \cdot \Lambda_{QCD}^4\right), \quad (32)$$

where

$$N_{ir} \approx \frac{2}{b_0 \alpha_s}$$

and terms proportional to  $\Lambda_{QCD}^4$  correspond to  $\langle 0 | (G_{\mu\nu}^a)^2 | 0 \rangle_{soft}$  which enters the QCD sum rules, for review see<sup>32</sup>.

A conspicuous feature of the prediction (32) is the absence of a quadratic correction, compare (11). Thus, we are seemingly coming to a contradiction between the lattice branes and continuum-theory perturbation theory. Let us, however, have a closer look at the problem.

### 5.2. Numerical results

Numerically, this perturbative expansion for the gluon condensate was studied in greatest detail in the papers in Ref <sup>14</sup> (especially in the latest one). The results can be summarized in the following way. On the lattice, the gluon condensate is nothing else but the average plaquette action. Represent the plaquette action  $\langle P \rangle$  as:

$$a^4 \langle P \rangle \approx P_{pert}^N + b_N a^2 \Lambda_{QCD}^2 + c_N a^4 \Lambda_{QCD}^4, \quad (33)$$

where the average plaquette action  $\langle P \rangle$  is measurable directly on the lattice and is known to high accuracy,  $P_{pert}^N$  is the perturbative contribution calculated up to order  $N$ :

$$P_{pert}^N \equiv 1 - \sum_{n=1}^{n=N} p_n g^{2n}, \quad (34)$$

and, finally coefficients  $b_N, c_N$  are fitting parameters whose value depends on the number of loops  $N$ . Moreover, the form of the fitting function (33) is rather suggested by the data than imposed because of theoretical considerations.

The conclusion is that up to ten loops,  $N = 10$  it is the quadratic correction which is seen on the plots while  $c_N$  are consistent with zero. However, the value of  $b_N$  decreases monotonically with growing  $N$  <sup>14</sup>. The factorial divergence (30) is not seen yet and perturbative series reproduces the measured plaquette action at the level of  $10^{-3}$ . Finally, at the level  $10^{-4}$  the  $\Lambda_{QCD}^4$  term seems to emerge <sup>14</sup>.

### 5.3. Implications

Thus, there is a fundamental difference between the instantons and branes. The instantons correspond to the ‘soft’ gluon condensate and are hidden in the  $(\Lambda_{QCD} \cdot a)^4$  corrections which are not calculable perturbatively. In short, instantons are added to perturbation theory.

The branes, on the other hand, appear to be dual to long perturbative series. If one is able to calculate many orders of perturbation theory, there is no need to account for the branes as far as *local* quantities are concerned.



This might first sound disappointing for those who is beginning to believe in the important role of the lattice branes. In fact it is not disappointing at all. To the contrary, we have first firm piece of evidence that the branes belong to a dual world. To reiterate: the instantons belong to the ‘direct’ formulation and they are added to the perturbation theory of the direct formulation. Branes belong to the dual formulation. Adding them to the perturbation theory of the direct formulation would be mixing up two different (dual to each other) representations of the same theory.

Actually, the very existence of fine tuned branes could not be understood within the direct formulation of the YM theory. However, if there were an existence theorem for a dual formulation, the fine (or self-) tuning was implied by the theorem. Reversing the argument, we can say that observation of the fine tuning might indicate existence of a dual formulation.

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